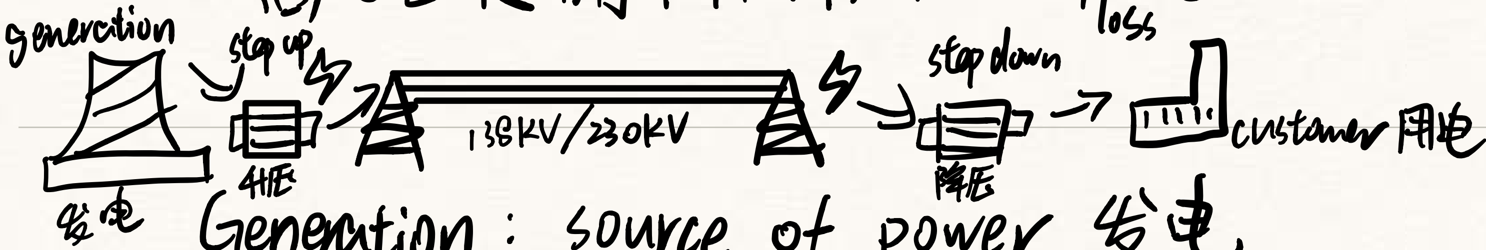
书《Power System Analysis and Design》

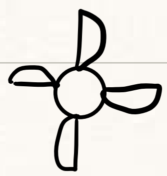
高电压传输降低损耗 $P_{loss} \propto i^2R$



Generation: source of power 发电

Load: consumes power 负载

Transmission system: transmits power 传输



Turbine 涡轮机

- Gas combustion 内燃 CT
- steam 蒸气 ST
- combined cycle 组合循环 CCT

可再生能源 renewable energy

Solar 太阳
- photovoltaic 光伏
- Thermal 热
- Concentrated 浓缩

turbine 涡轮

wind 风 → AC power { gear box 变速箱
power electronic converters AD转换

Geothermal Power 地热能

Tidal Power 潮汐能

Biomass 生物质能

power (watts) 瓦特　　K 10^3

$$P(t) = v(t) \cdot i(t)$$

M 10^6

G 10^9

Energy (Joules) 焦耳

$1J = 1W \cdot second(s)$

$1kWh = 3.6 \times 10^6 J$ 千瓦时

1 Btu (British Thermal Unit) = 1055J 英热

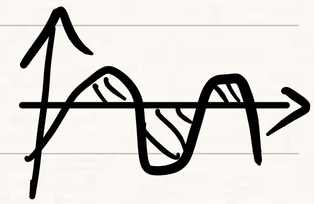
1 MBtu = 0.292 MWh

电力系统 {
AC/DC 交直流
60Hz 美 50Hz 其它
220V 中 380V 工业 110V 英美加 ...

以上内容了解即可

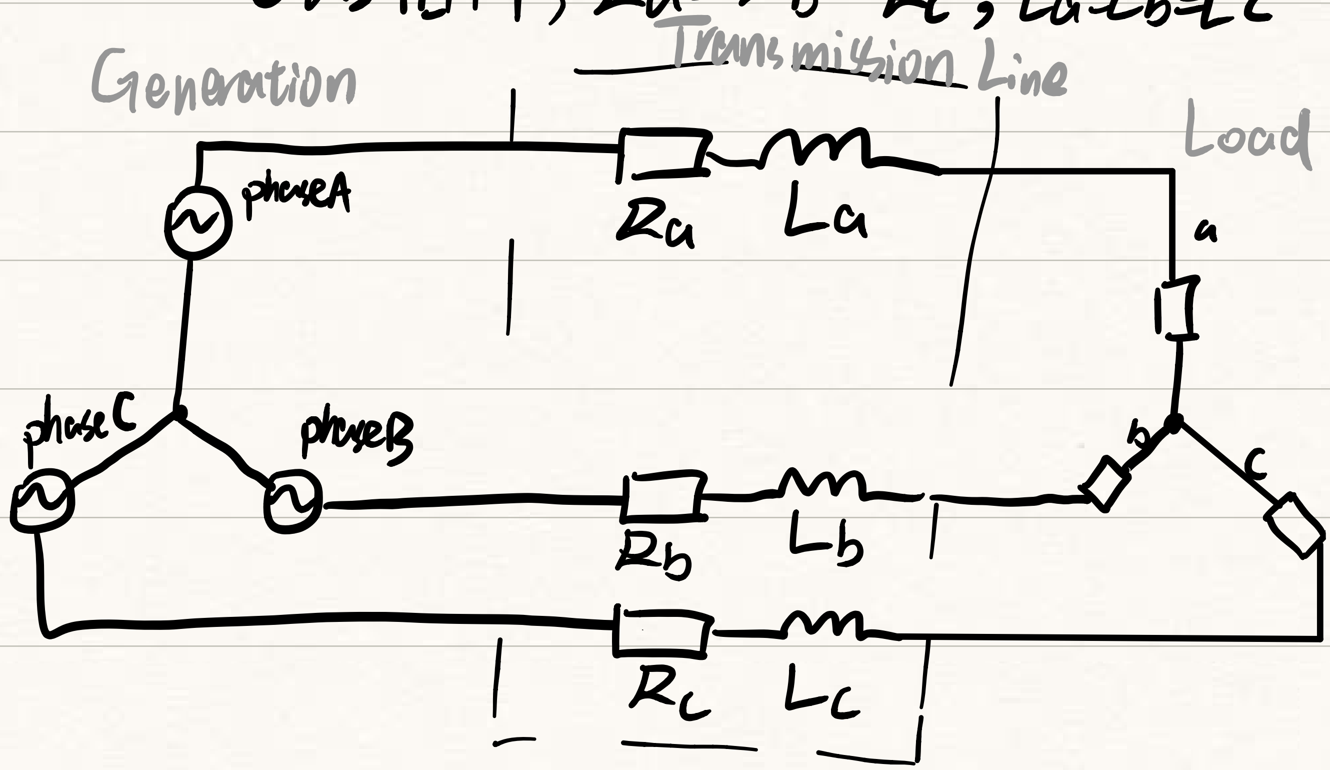
平均值 Average

AC交流 $V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt = 0$



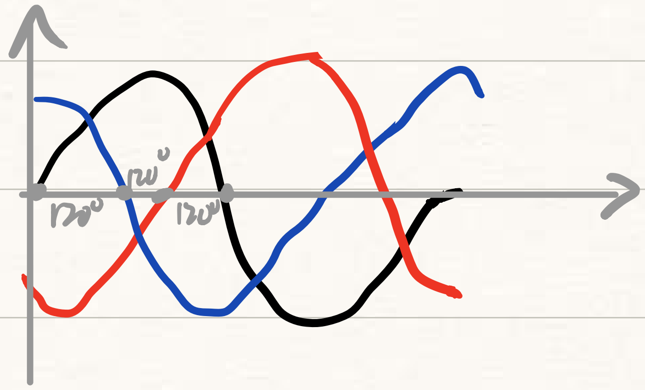
三相电压 Three phase power system

3路相同，$R_a = R_b = R_c$，$L_a = L_b = L_c$



v & i are 3 phase balanced

$P_{total} = P_a + P_b + P_c = 3P_a$ 不变量 constant



定义 $i_a(t) = A\sin(\omega t)$

$i_b(t) = A\sin(\omega t - 120°)$

$i_c(t) = A\sin(\omega t + 120°)$

instantaneous value 瞬时值 V_{max}

Root mean square 有效值，均方根

$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 dt}$, $v = V_{max}\cos\omega t$ ω角频率 angular frequency

$$= \sqrt{\frac{V_{max}^2}{T}\cdot\int_0^T \cos^2\omega t\, dt} = \frac{V_{max}}{\sqrt{T}}\cdot$$

$$\sqrt{\int_0^T \frac{\cos 2\omega t + 1}{2} dt} = \sqrt{\int_0^T \frac{\cos 2\omega t}{2} dt + \int_0^T \frac{1}{2} dt}$$

$$= \sqrt{\frac{\sin 2\omega t}{4\omega}\Big|_0^T + \frac{1}{2}t\Big|_0^T} = \sqrt{\frac{\sin 2\omega T}{4\omega} + \frac{T}{2}}$$

$$= \frac{V_{max}}{\sqrt{T}}\cdot\frac{\sqrt{T}}{\sqrt{2}} = \frac{V_{max}}{\sqrt{2}}$$

欧拉公式 Euler's identity 共轭 conjugate

$$e^{j\theta} = \cos\theta + j\sin\theta$$

直角坐标

$z = a+bj = \mu e^{j\theta}$

$z^* = a-bj = \mu e^{-j\theta}$

$$A\cos(\omega t+\theta) = \frac{A}{\sqrt{2}} \cdot e^{j\theta} = \frac{A}{\sqrt{2}} \angle\theta$$

相乘 向量 角度相加

$$Z_1 \cdot Z_2 = \mu_1 \mu_2 \cdot e^{j(\theta_1+\theta_2)}$$

相除 向量 角度相减

$$\frac{Z_1}{Z_2} = \frac{\mu_1}{\mu_2} \cdot e^{j(\theta_1-\theta_2)}$$

US 频率 $= 60Hz$ CN 频率 $f = 50Hz$

$\omega = 2\pi f = 377\,rad/s$

电阻 Resistor $V = RI$

电感 Inductor $V = j\omega LI$

电容 Capacity $V = \frac{1}{j\omega}$

阻抗 Impedance R, I, C 的总称 $Z = (R) + (C) + (L)$

导纳 Admittance 阻抗的倒数 $Y = \frac{1}{Z}$ 单位 Siemens (西门子)

Capacitor 设 $V(t) = V_{max}\cos(wt+\theta)$

在电容线路中， $i(t) = C\frac{dV(t)}{dt} = C\cdot w\cdot V_{max}\cdot\{-\sin(wt+\theta)\}$

$$= C w V_{max}\cos(wt+\theta+90^\circ)$$

$$Z_C = \frac{V(t)}{i(t)} = \frac{\frac{V_{max}}{\sqrt{2}}\angle\theta}{Cw\frac{V_{max}}{\sqrt{2}}\angle\theta+90^\circ} = \frac{1}{Cw}\angle-90^\circ = \frac{1}{jwC}$$

i 相位比V 快90°

current phasor leads the voltage by 90°

Indicator 设 $i(t) = I_{max}\cos(wt+\theta)$

在电感电路中 $V(t) = L\frac{di(t)}{dt} = L w I_{max}\cdot\{-\sin(wt+\theta)\}$

$$= L w I_{max}\cos(wt+\theta+90^\circ)$$

$$Z_L = \frac{V(t)}{i(t)} = \frac{\frac{LwI_{max}\angle\theta+90^\circ}{\sqrt{2}}}{\frac{I_{max}}{\sqrt{2}}\angle\theta} = Lw\angle 90^\circ = jwL$$

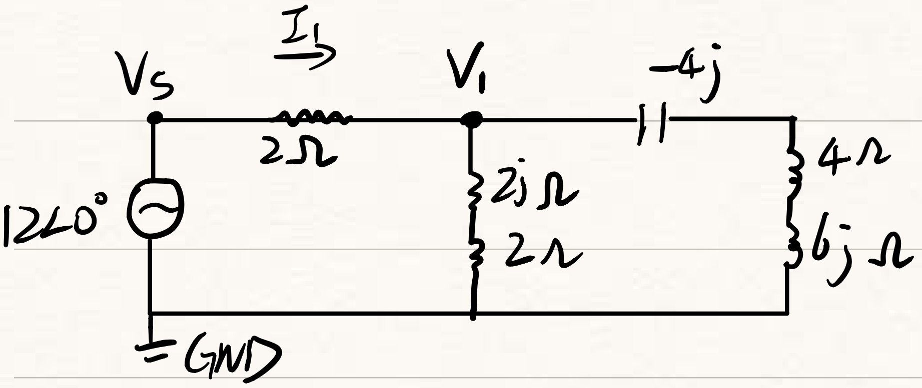
i 相位比 V 慢90°

current phasor lags the voltage by 90°

基尔霍夫定律 Kirchhoffs Circuit Law

KCL：结节电流之和(进,出)为零

KVL：闭路电压之和为零



KCL: $$\frac{V_1-V_s}{2}+\frac{V_1-0}{2+2j}+\frac{V_1-0}{4+6j-4j}=0$$

$$\Rightarrow V_1\left[\frac{1}{2}+\frac{1}{2+2j}+\frac{1}{4+6j-4j}\right]=\frac{V_s}{2} \quad (V_s=12\angle 0°)$$

$$\Rightarrow V_1=5.93\angle 20.2° \ (V)$$

功率 Power

电阻Resist $P(t)=V(t)\cdot i(t)=V_{max}I_{max}\cos^2(\omega t+\theta)$ W

$=\frac{1}{2}V_{max}I_{max}\{1+\cos(2\omega t+2\theta)\}$ W

$=\frac{1}{2}\cdot\frac{V_{max}^2}{R}\{1+\cos(2\omega t+2\theta)\}$ W

$V=\frac{V_{max}}{\sqrt{2}} \Rightarrow \frac{V^2}{R}+\frac{V^2}{R}\cos(2\omega t+2\theta)$ W

Indicator power $p(t)=V_{max}\cos(\omega t+\theta)I_{max}\cos(\omega t+\theta-90^\circ)$

$=\frac{1}{\omega L}V^2\sin(2\omega t+2\theta)$

Capacitor power $p(t)=\omega C V^2\sin(2\omega t+2\theta)$

电感电容两倍频率

$\omega_L, \omega_C = 2\omega$

RLC电路中

$\theta=\theta_V-\theta_I$

real 实

average power 平均功率: $P=\frac{V_{max}I_{max}}{2}\cos\theta$ 消耗的功率

imaginery 虚

reactive power 无用功率 $Q=\frac{V_{max}I_{max}}{2}\sin\theta$ 来回流动的能量

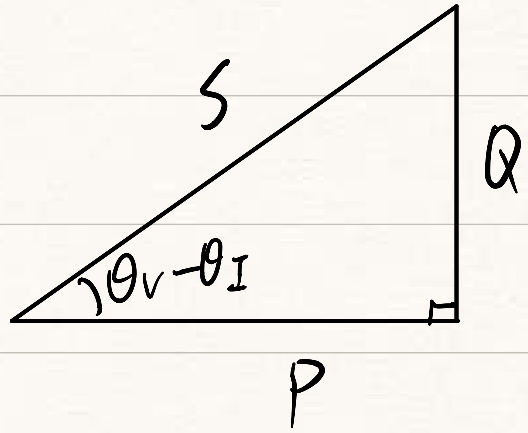
与相位差有关

power factor 功率因数 $PF=\cos(\theta_V-\theta_I)$

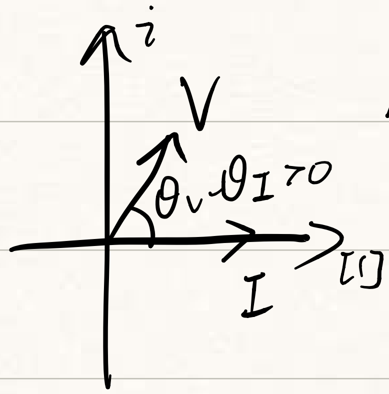
Complex power 复功率 $S=P+jQ$ VA

$P=\text{real}\{S\}$ W $=VI^*$ 共轭, $\left[\begin{array}{l} |V|\angle\theta_V, |I|\angle\theta_I \\ |V||I|\underbrace{\cos(\angle\theta_V-\theta_I)}_{e^{j(\theta_V-\theta_I)}} \end{array}\right]$

$Q=\text{img}\{S\}$ VAR



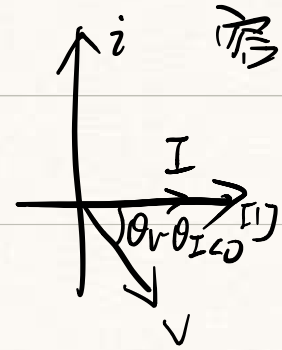
$PF = \cos(\theta_V - \theta_I) = \frac{P}{S}$



感性 j

V领先I θ相位

current lagging



容性 -j

I领先V θ相位

current leading

R $\quad S_R = V I_R^* = \frac{V^2}{R}$

L $\quad S_L = V \cdot I_L^* = j\frac{V^2}{X_L}, \; j\frac{V^2}{\omega L}$

C $\quad S_C = V \cdot I_C^* = -j\frac{V^2}{X_C}, \; -j\omega C V^2$

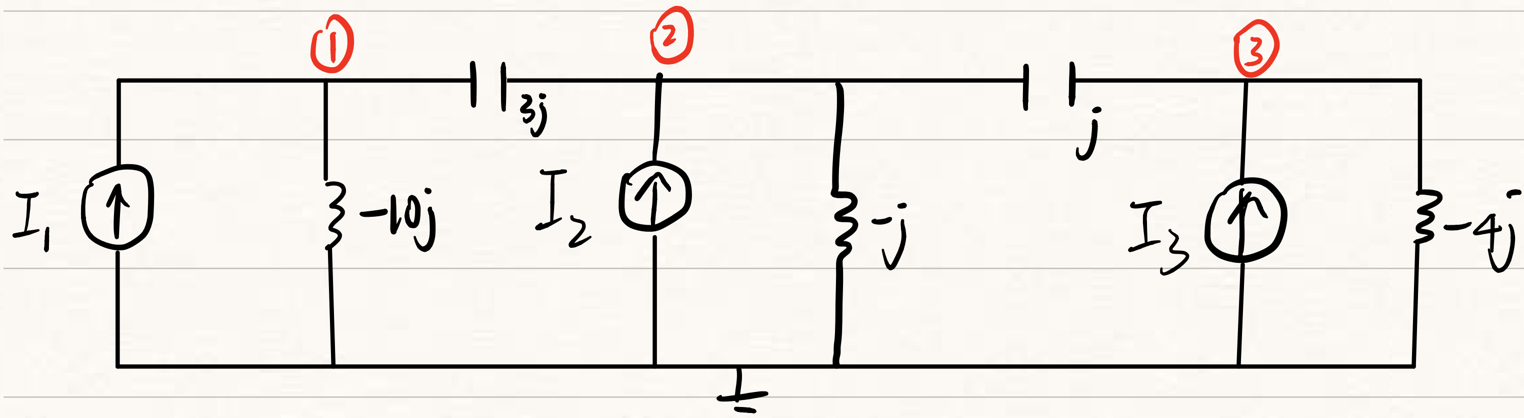
Apparent Power 视在功率 $|S| = \sqrt{P^2+Q^2} = |V||I|$

Power factor correct or reactive compensation

功率因素校正 无功补偿

在电路中，Pf 越近1越高效

nodal analysis 节点电流法 KCL 解复杂电路



导纳 admittances $Y = \frac{1}{Z}$ (S) 解方便计算

已知 $I_{1,2,3}$ 求 $V_{1,2,3}$

①: $(V_1 - V_2)\cdot 3j + (V_1 - 0)(-10j) - I_1 = 0$

②: $(V_2 - V_1)\cdot 3j + (V_2 - 0)(-j) + (V_2 - V_3)j - I_2 = 0$

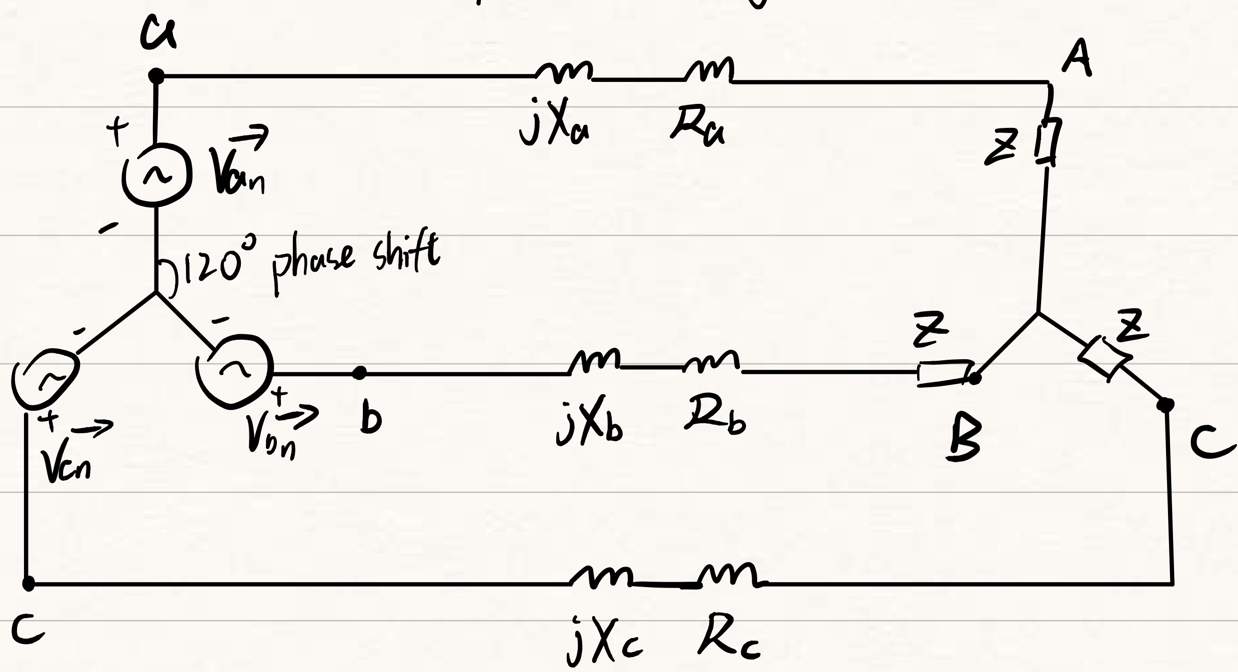
③: $(V_3 - V_2)\cdot j + (V_3 - 0)(-4j) - I_3 = 0$

$$\Rightarrow \begin{cases} I_1 = -7jV_1 - 3jV_2 \\ I_2 = -3jV_1 + 3jV_2 - 1jV_3 \\ I_3 = -1jV_2 - 3jV_3 \end{cases}$$

$$\Rightarrow \underbrace{\begin{bmatrix} -7j & -3j & 0 \\ -3j & 3j & -j \\ 0 & -j & -3j \end{bmatrix}}_{Y_{bus}:\ \text{Bus admittance matrix}} \underbrace{\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}}_{V_{bus}} = \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}}_{I_{bus}}$$

$$Y_{bus} \cdot V_{bus} = I_{bus}$$

Three phrase system 三相电路

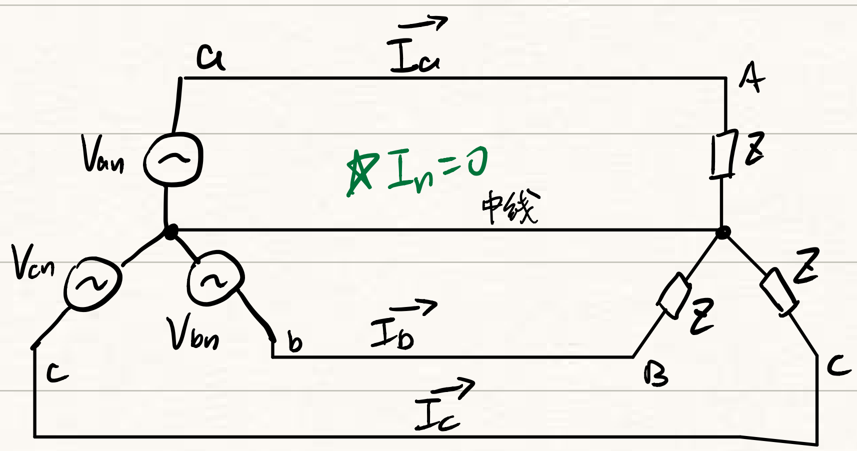


☆ $\vec{V}_{an}=|V_{an}|\angle 0°$, $\vec{V}_{bn}=|V_{bn}|\angle -120°$, $\vec{V}_{cn}=|V_{cn}|\angle 120°$, $R_a=R_b=R_c$

define $a=1\angle 120°$ $V_{bn}=a^2V_{an}$ $I_{bn}=a^2I_{an}$

$V_{cn}=aV_{an}$ $I_{cn}=aI_{cn}$

Y型连接电路 wye(Y)connection



in balance system 平衡系统中
no neutral current 无中性流,
I总和为零

$$I_n=I_a+I_b+I_c=\frac{V}{Z}(1\angle 0°+1\angle -120°+1\angle 120°)=0$$

$$S=V_{an}I_{an}^*+V_{bn}I_{bn}^*+V_{cn}I_{cn}^*=3V_{an}I_{an}^*$$

3 phase complex Power $S_{3\phi}$ 三相功率

$$S_{3\phi} = S_A + S_B + S_C = V_{an} I_a^* + V_{bn} I_b^* + V_{cn} I_c^*$$

$$= V_{an} I_a^* + (a^2 V_{an}) \cdot (a^2 I_a)^* + (a V_{an})(a I_a)^*$$

$$= V_{an} I_a^* + a^2 \cdot (a^2)^* V_{an} I_a^* + a \cdot a^* V_{an} I_a$$

$$= V_{an} I_a^* (1 + \underbrace{|a|^4}_{1} + \underbrace{|a|^2}_{1})$$

$$= 3 V_{an} I_a^*$$

Phase Voltage: Voltage across each load (source) in each phase

三相(负载)电压 $\begin{cases} V_{an}, V_{bn}, V_{cn} \\ V_{AN}, V_{BN}, V_{CN} \end{cases}$ 三相负载上电压

phase current: current through load in each phase

三相(负载)电流 $\begin{cases} I_a, I_b, I_c \\ \end{cases}$ 三相负载上电流

Line voltage (line to line Voltage): voltage difference between 2 phases

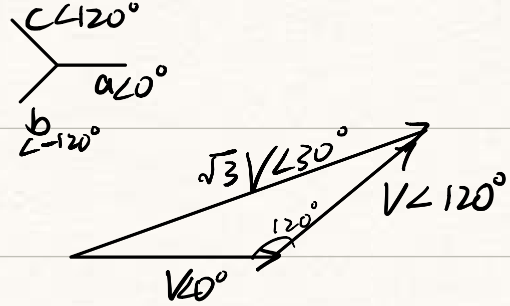
(传输间) 线电压 $\begin{cases} V_{ab} = V_{an} - V_{bn} \\ V_{bc} = V_{bn} - V_{cn} \\ V_{ac} = V_{an} - V_{cn} \end{cases}$ 传输线间相位电压差

Line current: currents through transmission line in each phase

(传输) 线电流 $\begin{cases} I_a, I_b, I_c \end{cases}$ 传输线上电流

Y 线电流 = 三相电流

Y线电压 Y connection - Line Voltage

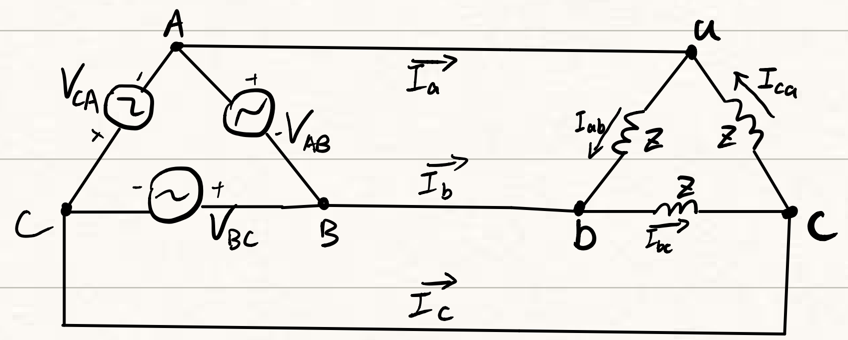


$$V_{ab} = V_{an} - V_{bn} = |V|(1\angle 0^\circ - 1\angle -120^\circ)$$
$$= \sqrt{3} V_{an} \angle 30^\circ$$

$$V_{bc} = \sqrt{3} V_{bn} \angle 30^\circ = \sqrt{3} V_{an} \angle 30^\circ - 120^\circ = \sqrt{3} V_{an} \angle -90^\circ$$

$$V_{ca} = \sqrt{3} V_{cn} \angle 30^\circ = \sqrt{3} V_{an} \angle 30^\circ + 120^\circ = \sqrt{3} V_{an} \angle 150^\circ$$

三角型电路 Delta(Δ) connection



phase voltage 三相(负载)电压

(V_{AB}、V_{BC}、V_{CA}) & (V_{ab}、V_{bc}、V_{ca})

电压 负载

Δ型 三相电压=线电压

line-line voltage (传输间) 线电压

(V_{AB}、V_{BC}、V_{CA}) & (V_{ab}、V_{bc}、V_{ca})

电压 负载

phase currents 三相(负载)电流

(I_{ab}, I_{bc}, I_{ca})

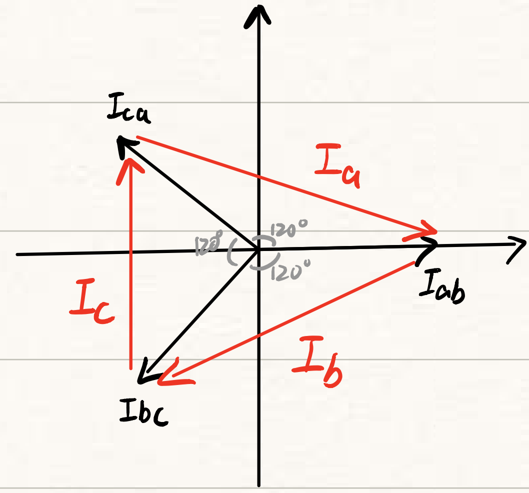
line currents (传输)线电流

(I_a, I_b, I_c)

用KCL得 $I_a = I_{ab} - I_{ca}$

$I_b = I_{bc} - I_{ab}$

$I_c = I_{ca} - I_{bc}$



$I_a = \sqrt{3} I_{ab} \angle -30^\circ$

$I_b = \sqrt{3} I_{bc} \angle -30^\circ$

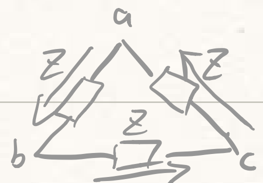
$I_c = \sqrt{3} I_{ca} \angle -30^\circ$

$S_{3\varphi} = 3 V_{phase} I_{phase}^*$

例 Δ-connected load with $Z = 120\angle 30^\circ \Omega$ supplied by 3ϕ 12kV (L-L) source. Find complex power in each phase of load

一定是有效值rms

$V_{ab} = 12kV\angle 0^\circ$ set as ref



$S_a = V_{ab} I_{ab}^*$ Δ三相电压=线电压，求负载功率用负载三相电流

$I_{ab} = \dfrac{V_{ab}}{Z} = \dfrac{12kV\angle 0^\circ}{120\angle 30^\circ} = 100\angle -30^\circ \, (A)$

$\begin{cases} I_{bc} = 100\angle -30^\circ - 120^\circ = 100\angle -150^\circ \\ I_{ca} = 100\angle -30^\circ + 120^\circ = 100\angle 90^\circ \end{cases}$

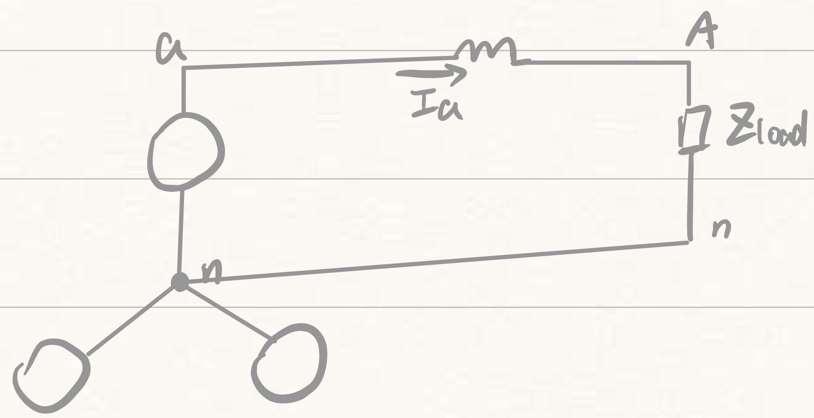
$S_a = V_{ab} \cdot I_{ab}^* = 1200k\angle 30^\circ$

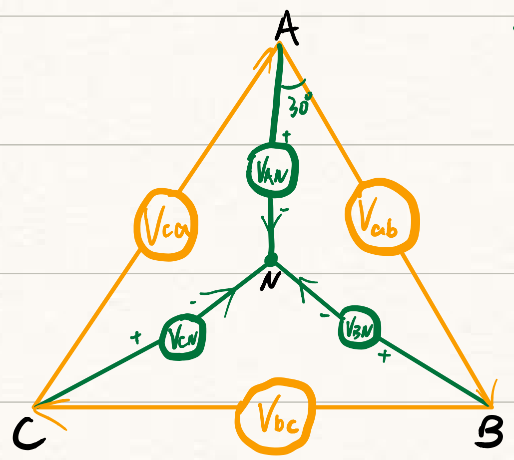
和$I_{a,b,c}$

$\begin{cases} I_a = \sqrt{3} I_{ab} \angle -30^\circ = 100\sqrt{3}\angle -60^\circ \ A \\ I_b = \sqrt{3} I_{bc} \angle -30^\circ = 100\sqrt{3}\angle -180^\circ \ A \\ I_c = \sqrt{3} I_{ca} \angle -30^\circ = 100\sqrt{3}\angle 60^\circ \ A \end{cases}$

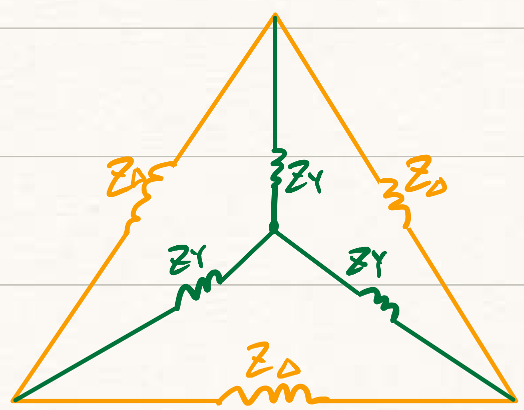
Y−Δ conversion 变化

分析 3相系统时，为简化，只分析其中一条路线，其余适用





$$V_{AN} = \frac{V_{ab}}{\sqrt{3}}\angle -30^\circ$$

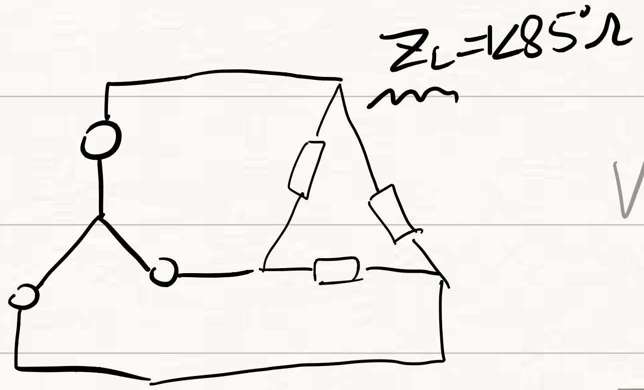


$$Z_Y = \frac{\frac{V_{phase}}{\sqrt{3}}\angle -30^\circ}{I_{phase}\sqrt{3}\angle -30^\circ} = \frac{Z_\Delta}{3}$$

例 已知 Y型电压 $V_{ab}=480\angle 0^\circ V$

△型负载 $Z_\Delta = 30\angle 40^\circ \Omega$，line impedance 线阻抗

为 $1\angle 85^\circ \Omega$，求线电流，△负载电流，电压



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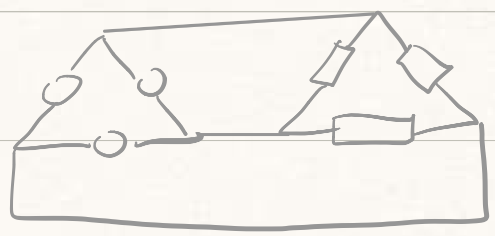
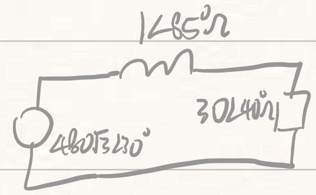


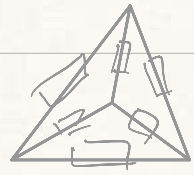
$$V_{ab}^{\Delta} = \sqrt{3}\, V_{ab}^{Y}\angle 30^\circ$$
$$= 480\sqrt{3}\angle 30^\circ$$



$$I_a = \frac{V_{ab}^{\Delta}}{1\angle 85^\circ + 30\angle 40^\circ}$$

★所有给定三相问题电压都是两线差电压（line (to line) Voltage）而不是到中线电压差（phase Voltage）

例 三相 线负载 $2+4j\ \Omega$ 源 60Hz，$120\sqrt{3}$ V 电压 三相平衡了



1st Y型负载 $Z_{L_1}^{Y} = 30+40j\ \Omega$

+

2nd △型负载 $Z_{L_2}^{\Delta} = 60-45j\ \Omega$ $\quad Z_{L_2}^{\Delta} \xrightarrow{Y} Z_{L_2}^{Y} = \frac{Z_L^{\Delta}}{3} = 20-15j\ \Omega$

1) 求电流，功率 real & reactive

assume $V_{AN} = \frac{120\sqrt{3}}{\sqrt{3}}\angle 0° = 120\angle 0°\ V$

$$I_a = \frac{V_{AN}}{Z_{Y_1} // Z_{Y_2}} = \frac{120\angle 0°}{24} = 5\angle 0°\ A$$

$S_s = V_{AN} \times I_a^{*} = 120\angle 0° \cdot 5\angle 0° = 600\ VA$, $P_s = 600W$, $Q = 0$

$S_s^{3\varphi} = 3S_s = 1800VA \quad P_s^{3\varphi} = 3P_s = 1800w$

three phase power instantaneous 三相功率瞬时值

$V_{an}(t) = \sqrt{2} V_{LN} \cos(\omega t + \delta)$ V

$i_a(t) = \sqrt{2} I_L \cos(\omega t + \beta)$ A

$2\cos a \cos b = \cos(a+b) + \cos(a-b)$

$$P_a(t) = V_{an}(t) \cdot i_a(t) = 2 V_{LN} I_L \cos(\omega t + \delta) \cos(\omega t + \beta) \text{ w}$$

$$= V_{LN} I_L \cos(\delta - \beta) + V_{LN} I_L \cos(2\omega t + \delta + \beta) \text{ w}$$

$$P_{3\varphi}(t) = P_a(t) + P_b(t) + P_c(t)$$

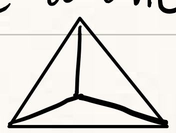
$$= 3 V_{LN} I_L \cos(\delta - \beta) + V_{LN} I_L \left[\begin{aligned} &\cos(2\omega t + \delta + \beta) \\ &+ \cos(2\omega t + \delta + \beta - 240^\circ) \\ &+ \cos(2\omega t + \delta + \beta + 240^\circ) \end{aligned} \right]$$

LN

line to neuture $P_{3\varphi}(t) = 3 V_{LN} I_L \cos(\delta - \beta)$ W 三相电压

LL

Line to line $P_{3\varphi}(t) = \sqrt{3} V_{LL} I_L \cos(\delta - \beta)$ w 线电压



三相电压瞬时值为常数

磁场H、B计算

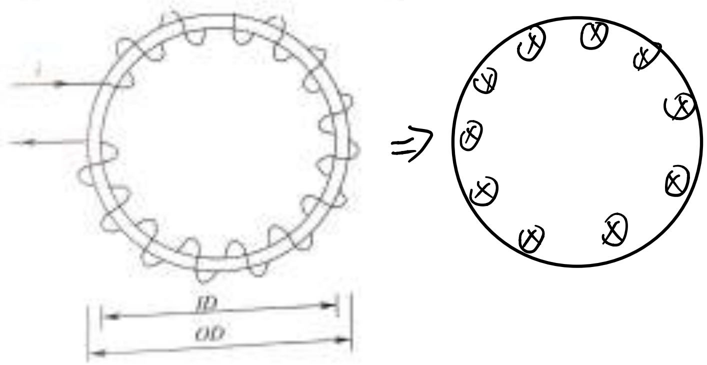
安培定律 Ampere's Law

$$\oint H dl = \sum I_{enc}$$

例：

Example:

Consider a coil which has N=25 turns and the toroid has an inside diameter of I_D=5cm and outside diameter of O_D=5.5 cm. For a current i=3A, calculate the field intensity *H* along the mean-path length within the toroid.



$d = \frac{I_D + O_D}{2} = 5.25cm$, $\oint H dl = NI$,

$2\pi r_m = 0.165\ m$ $H = \frac{NI}{2\pi r_m} = \frac{25\times3}{0.165m} = 454.5\ A/m$

磁感应强度 $B=\mu_0 H$ $\quad(\mu_0=4\pi\times10^{-7}\ H/m)$

(又叫磁通量密度) B: flux density 单位 Tesla 特斯拉

磁通量 $\Phi=\int_s B\,ds=BA$ (垂直面积)

Φ: flux 单位 Wb 韦博

ferromagetism 强磁性，μ 仅在小区间呈线性

磁通势 Magnetomotive force $mmf=Hl=\Phi\cdot R=NI$

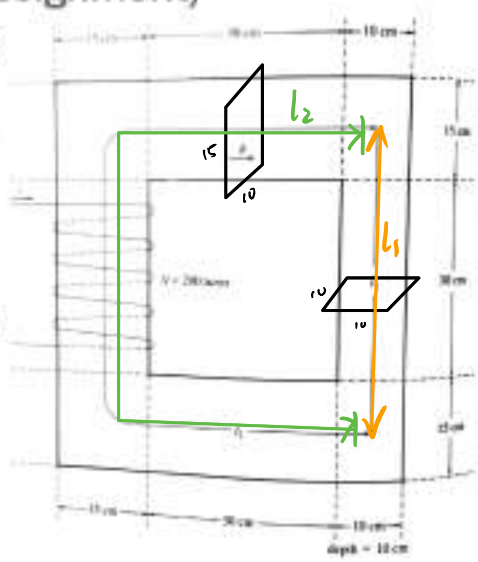
$$\begin{cases} B=\mu H \\ \Phi=B\cdot \underset{\text{面积}}{A} \end{cases} \Rightarrow mmf=\Phi\cdot\boxed{\frac{l}{\mu A}}$$

定义 reluctance 磁阻 $R=\frac{l}{\mu A}$

例:

Example 1 (class assignment)

A ferromagnetic core has three sides of uniform width and a thinner side. The depth of the core (into the page) is 10 cm, and other diameters are as shown. This is a 200-turn coil wrapped around the left side of the core. Assuming μr=2500, how much flux will be produced by 1 (A) current?



$$l_1 = 30 + \frac{15}{2} + \frac{15}{2} = 45\ cm$$

$$R_1 = \frac{l_1}{\mu A_1} = \frac{60 \times 10^{-2} m}{2500 \times 4\pi \times 10^{-7} \times 0.1^2 m^2} = 14323.9$$

$$l_2 = 30 + \frac{15}{2} + 30 + \frac{15}{2} + \frac{15}{2} + 30 + \frac{15}{2} + \frac{10}{2} + \frac{10}{2}$$
$$= 130cm$$

$$A_2 = 15 \times 10 = 0.015 m^2$$

$$R_2 = \frac{l_2}{\mu A_2} = \frac{130 \times 10^{-2} m}{2500 \times 4\pi \times 10^{-7} \times 0.015 m^2} = 27586.9$$

reluctance are in serial

$$\Phi = \frac{NI}{R_{total}} = \frac{200 \times 1A}{R_1 + R_2} = 0.0048\ wb$$

flux linkage 磁链 $\lambda = N \cdot \Phi = Li$

$$\left\{ \begin{array}{l} L = \frac{\lambda}{i} \\ \lambda = N\Phi \\ \Phi = \frac{Ni}{R} \end{array} \right. \Rightarrow L = \frac{N^2}{R}$$

N coil

Faraday's Law 法拉第电磁感应定律

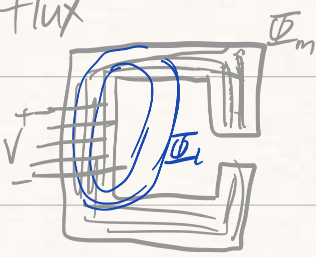
变化Φ磁通量产生电压

$$V = \frac{d\lambda}{dt} = \frac{dN\Phi}{dt}$$

Lenz's Law 楞次定律

感应电流的磁场总要阻碍磁场变化

leakage flux
磁泄露

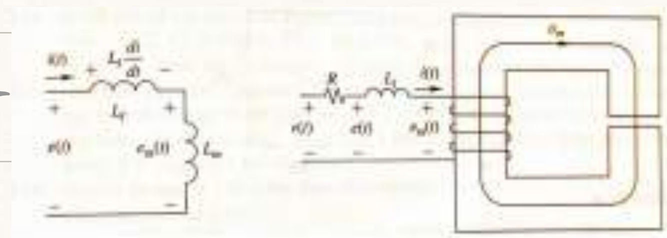


$$\Phi = \underbrace{\Phi_m}_{\text{magnet flux}} + \underbrace{\Phi_l}_{\text{leakagefux}}$$

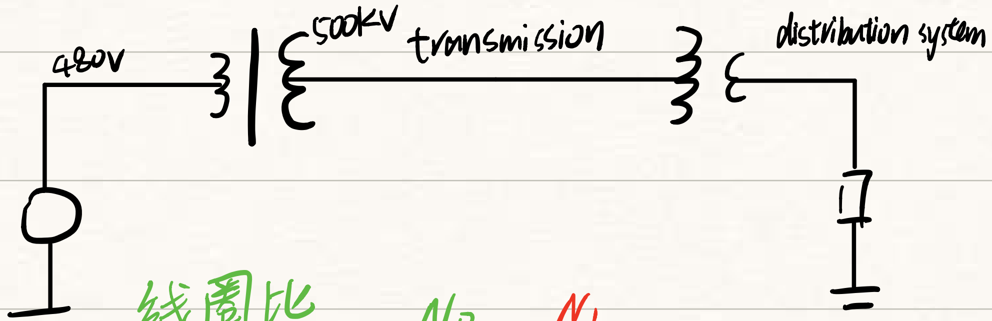
$$\lambda = N\Phi = \lambda_m + \lambda_l$$

$$L = \frac{\lambda}{i} = \underbrace{L_m}_{\text{maget inductance}} + \underbrace{L_l}_{\text{leakage inductance}}$$

算入磁泄露，$V = L_m \frac{di}{dt} + L_l \frac{di}{dt}$

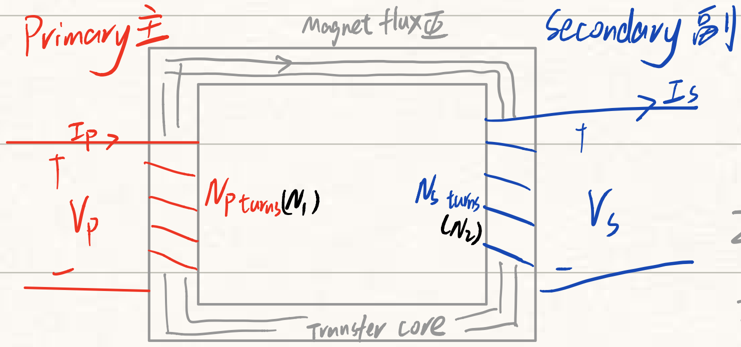


理想变压器 ideal transformer



线圈比

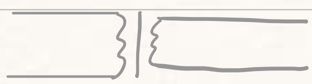
turn ratio $a_t = \frac{N_p}{N_s} = \frac{N_1}{N_2}$



理想变压器

1. 无功率损耗 no power loss $P_{in} = P_{out}$
2. 磁芯 μ 无限大 permeability infinite $\mu \to \infty$ $R \to 0$
3. 无磁泄露 no leakage flux

简写



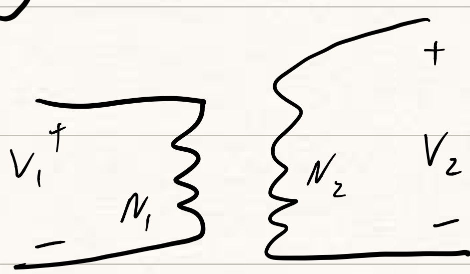
$$N_1 I_1 = N_2 I_2 = Hl$$

$$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

Secondary impedance referred to primary side:

副线圈在主电路的等效电阻

known $\begin{cases} \frac{V_1}{V_2} = \frac{N_1}{N_2} \\ \frac{I_1}{I_2} = \frac{N_2}{N_1} \end{cases}$, $P_1 = P_2$



$$Z_{2ref} = \frac{V_1}{I_1} = \frac{aV_2}{\frac{1}{a}I_2} = a^2 Z_2$$

$$= \left(\frac{N_1}{N_2}\right)^2 Z_2$$

Example 3

- Calculate the impedance of the load on the secondary of an ideal transformer and its value referred to the primary side.

N_1=2000; N_2=500; $V_1=1200\angle 0°(V)$; $I_1=5\angle -30°(A)$;

$$\begin{cases} \dfrac{V_1}{V_2} = \dfrac{N_1}{N_2} \\ \dfrac{I_1}{I_2} = \dfrac{N_2}{N_1} \end{cases}, \text{ so } \quad \begin{aligned} V_2 &= \frac{N_2}{N_1} \cdot V_1 = 300\angle 0° \, V \\ I_2 &= \frac{N_1}{N_2} \cdot I_2 = 20\angle -30° (A) \end{aligned}$$

$$Z_2 = \frac{V_2}{I_2} = 15\angle 30° (\Omega)$$

$$Z_2' = Z_2 \cdot \left(\frac{N_1}{N_2}\right)^2 = 16 Z_2$$

$$= 240\angle 30° (\Omega)$$

Practical transformer 实际变压器

计算实际磁损失 ($\Phi = \frac{NI}{R_{total}}$, E_1, E_2 线圈电压)

$$\begin{cases} N_1 I_1 - N_2 I_2 = \underline{R_c \Phi_c} \\ \Phi_c = \dfrac{E_1}{j\omega N_1} \end{cases}$$

$$\Rightarrow N_1 I_1 - N_2 I_2 = \frac{E_1 R_c}{j\omega N_1}$$

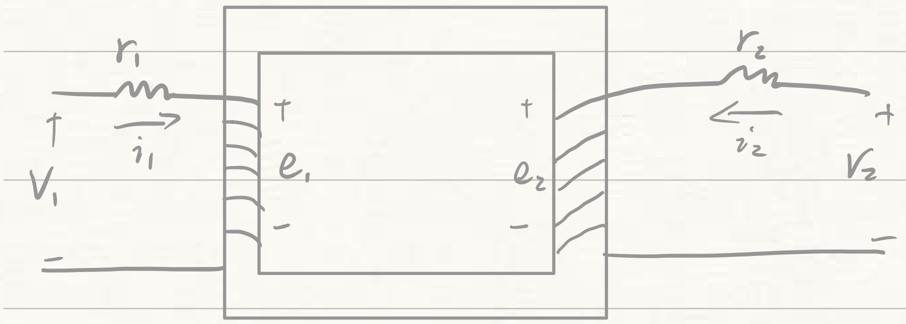
$$I_1 = \frac{N_2}{N_1} \cdot I_2 + \underline{\frac{R_c}{N_1}\left(\frac{E_1}{j\omega N_1}\right)}$$

$I_m = \dfrac{R_c E_1}{j\omega N_1^2}$ magnetizing current 磁化电流

$$= \frac{N_2}{N_1} \cdot I_2 - j\left(\frac{R_c}{j\omega N_1^2}\right) E_1$$

B_m 似电感、阻抗导纳

自感. 互感.



$v_1 = i_1 r_1 + e_1$, $v_2 = i_2 \cdot r_2 + e_2$

假设 i_1 acting alone

flux linkage $\lambda_{11} = N_1 \Phi_{11} = L_{11} i_1$

$\lambda_{21} = N_2 \Phi_{21} = L_{21} \cdot i_1$

假设 i_2 acting alone

$\lambda_{22} = N_2 \Phi_{22} = L_{22} \cdot i_2$

$\lambda_{12} = N_1 \Phi_{12} = L_{12} i_2$

self Inductance
自感系数

Mutual Inductance
互感系数 $L_{12} = L_{21}$

$$\Rightarrow \begin{cases} \lambda_1 = \lambda_{11} + \lambda_{12} \\ \lambda_2 = \lambda_{22} + \lambda_{21} \end{cases} \Rightarrow \begin{cases} \lambda_1 = L_{11} i_1 + L_{12} i_2 \\ \lambda_2 = L_{22} i_2 + L_{21} \cdot i_1 \end{cases}$$

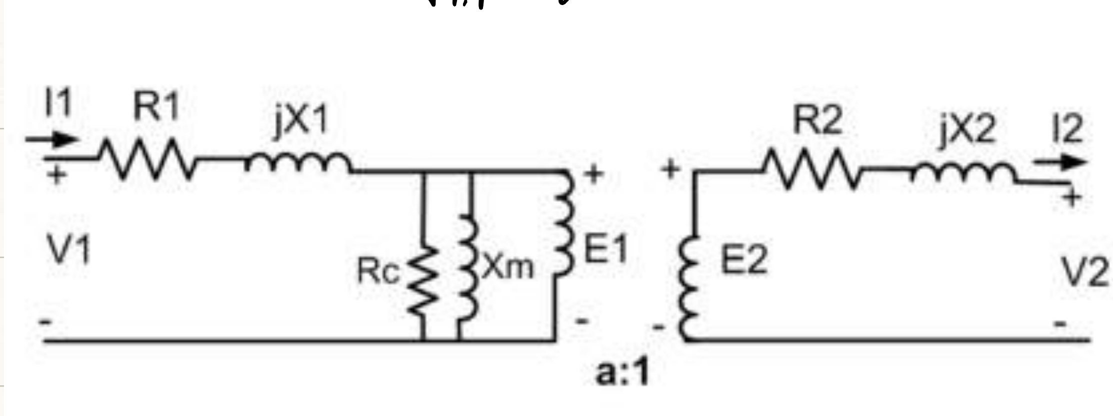
时域

$$\begin{cases} V_1 = i_1 r_1 + L_{11} \dfrac{di_1}{dt} + L_{12} \dfrac{di_2}{dt} \\ V_2 = i_2 r_2 + L_{22} \dfrac{di_2}{dt} + L_{21} \dfrac{di_1}{dt} \end{cases}$$

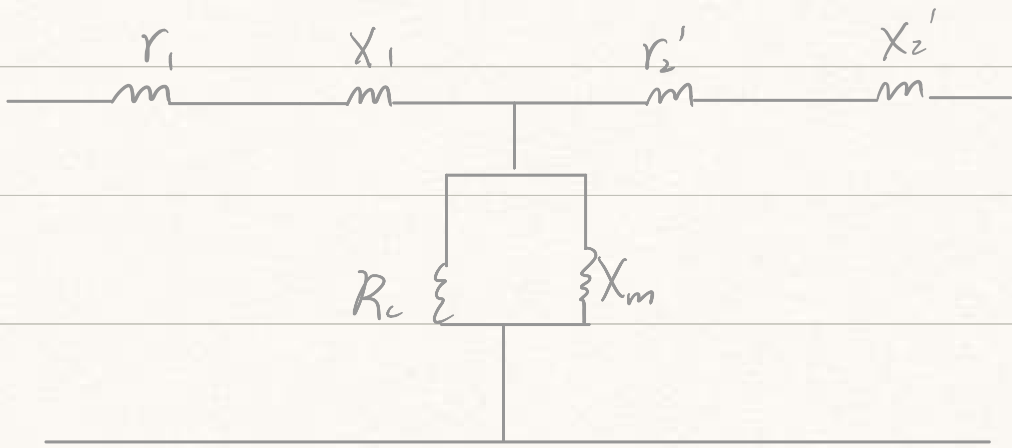
频域

$$\begin{cases} V_1 = \underbrace{(r_1 + jwL_{11})}_{Z_{11}} \cdot i_1 + \underbrace{jwL_{12}}_{Z_{12}} \cdot i_2 \\ V_2 = \underbrace{(r_2 + jwL_{22})}_{Z_{22}} \cdot i_2 + \underbrace{jwL_{21}}_{Z_{21}} \cdot i_1 \end{cases}$$

等价 equivalent circuit



简化变压器



$(a=\frac{N_1}{N_2})$

referred to the primary side

$r_2' = a^2 r_2 \qquad r_e = r_1 + r_2'$

$x_2' = a^2 x_2 \qquad x_e = x_1 + x_2'$

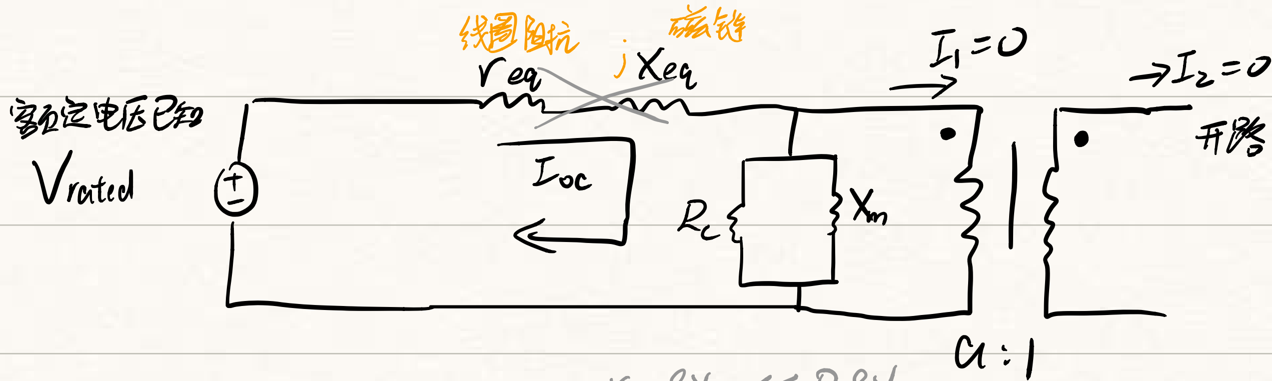
计算参数 parameters

① 开路 open circuit test

Xm 磁感应阻抗 magnet resistance

Rc 磁泄露 core' losses

副线圈断路求 I_1 和 P_{loss}



r_{eq} & $X_{eq} << R_c$ & X_m

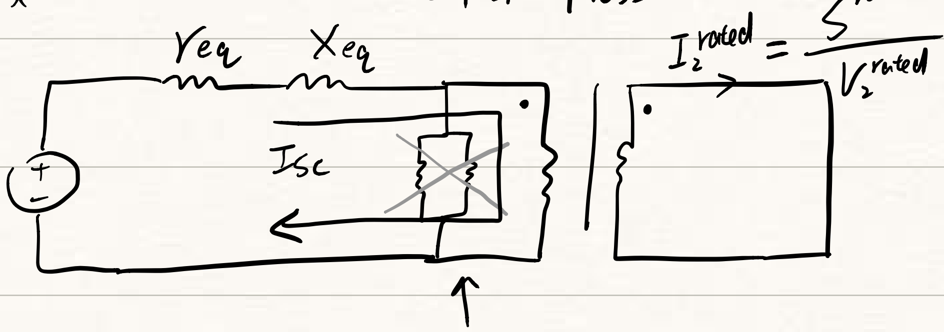
$$P_{loss}=\frac{V_c^2}{R_c} \quad \left(R_c=\frac{1}{G_c}\right) \qquad V_2=V_{rated}$$

$$I_{oc}=\frac{V_{rated}}{R_c+jX_m}$$

② 短路 short circuit test

副线圈短路测 V_{rated} 和 P_{loss}

$$I_2^{rated}=\frac{S^{rated}}{V_2^{rated}}$$



输入电压很小，所以电流在磁化分支很小，忽略不计

Magnetizing branch

$$P_{loss} = r_{eq} \cdot I_{sc}^2$$

$$Z_{eq} = r_{eq} + jX_{eq} = \frac{V_{sc}}{I_{sc}}$$ 可求 V_{sc}

Example: Model Parameter Calculation

A single phase, 100 MVA (S_{rated}), 200/80 kV (N_1 N_2) transformer has the following test data:

open circuit: 20 amps (I_{oc}), with 10 kW (P_{loss}^{oc}) losses

short circuit: 30 kV (V_{sc}), with 500 kW (P_{loss}^{sc}) losses

Find the model parameters.

例 open circuit test

$$V_{oc} = V_1^{rated} = 200kV$$

$$P_{loss} = \frac{V_{oc}^2}{R_c} = G_c \cdot V_{oc}^2$$

$$\Rightarrow G_c = 25\times10^{-8}, \quad R_c = 4M\Omega$$

Short circuit

$$I_{sc} = I_1^{rated} = \frac{S_{rated}}{V_1^{rated}} = \frac{100MVA}{200kV} = 500A$$

$$r_{eq} = \frac{P_{loss}}{I_{sc}^2} = \frac{500kW}{500A^2} = 2\Omega$$

$$|Z_{eq}| = \frac{V_{sc}}{I_{sc}} = \frac{30kV}{500A} = 60\Omega$$

$$X_{eq} = \sqrt{Z_{eq}^2 - r_{eq}^2} = 59.96\Omega$$

rated 额定电压，以实际值为准

Per unit system (pu.) 标幺值

$$\text{quantity per unit 标幺值} = \frac{\text{actual quantity 实际值}}{\text{base value of quantity 基准值}} \quad [pu.]$$

$$S_{pu} = \frac{S}{S_b}$$

怎么选基准值 Base value? (base缩写b小标, S_b, V_b)

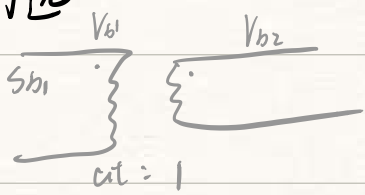
先选 S_b, V_b

$$I_b = \frac{S_b}{V_b}, \quad Z_b = \frac{V_b}{I_b} = \frac{V_b^2}{S_b^2}$$

X值 $X = 2\% = \frac{X^{actual}}{Z_{base}}$, $Z_{base} = \frac{V_{rated}^2}{S_{rated}}$ 假设 $S_b = S_{rated}$, $V_b = V_{rated}$

变压器标幺值按线圈比例关联 $\frac{V_{b1}}{V_{b2}} = \frac{N_1}{N_2} = a_t$, ($S_b$相同)

主副端可选



$$Z_{b1} = \frac{V_{b1}^2}{S_b}, \quad I_{b1} = \frac{S_b}{V_{b1}}$$

$$Z_{b2} = \frac{V_{b2}^2}{S_b} = \frac{1}{a_t^2} Z_{b1}, \quad I_{b2} = \frac{S_b}{V_{b2}} = a_t I_{b1}$$

bLV 即 base Low Voltage bHV 即 base High Voltage

$$Z_{bLV} = \frac{V_{bLV}}{I_{bLV}} = \frac{V_{bLV}^2}{S_b} \qquad Z_{bHV} = \frac{V_{bHV}}{I_{bHV}} = \frac{V_{bHV}^2}{S_b}$$

S_b 容功率保持不变

$$\begin{cases} S_{pu} = \dfrac{S}{S_b} = V_{pu} I_{pu}^* \\ P_{pu} = \dfrac{P}{S_b} = V_{pu} I_{pu} \cos\theta \\ Q_{pu} = \dfrac{Q}{S_b} = V_{pu} I_{pu} \sin\theta \end{cases}$$

例:

$Z\% = Z_{pu} \times 100\%$

leakage of transformer is 10%, given $X = 10\% = 0.1\ (pu.)$

$R_{eq} = 2\% = 0.02\ (pu.)$

则 $S_b = S_{rated} = 100MVA$

$V_{b1} = 100kV$

得 $Z_{b1} = \dfrac{V_{b1}^2}{S_b} = 100\Omega$

$R_{eq1} = Z_{b1} \cdot R_{eq}(pu.) = 2\Omega$

$X_{e1} = Z_{b1} \cdot X(pu.) = 10\Omega$

例: $V_{rated} = 120V$, $S_{rated} = 500\ VA$

It V applied across the load twice the rated value and load draws rated power. 算 I

1) choose base value

$V_b = 120V$

$S_b = 500VA$

$I_b = \dfrac{S_b}{V_b} = \dfrac{500}{120} = 4.167A$

2) convert to pu.

$$V_{load} = \frac{240}{120} = 2(pu.),\quad I_{load} = \frac{500}{V_{load}} = 0.5(pu.)$$

$$P_{load} = \frac{P}{S_b} = V_{pu} I_{pu} \cos\theta = 1\,pu.$$

如果原 S_b, V_b 不一致，反过来按比例算 Z_{pu} $\quad \frac{Z_{pu2}}{Z_{pu1}} = \frac{\frac{Z_{actual}}{Z_{b2}}}{\frac{Z_{actual}}{Z_{b1}}} = \frac{Z_{b1}}{Z_{b2}}$

'1' old '2' new

$$\Rightarrow Z_{pu2} = \frac{Z_{b1}}{Z_{b2}} \cdot Z_{pu1}$$

☆ 不同 base 转换 考 $\quad$ 1 old $\quad$ 2 new

$V_{pu} = \frac{V_{actual}}{V_{base}}$ 与 V_{base} 反比

$PQS_{pu} = \frac{PQS}{S_{base}}$ 与 S_{base} 反比

$ZXR_{pu} = \frac{V_{base}^2}{S_{base}}$ 与 $\frac{V_{base}^2}{S_{base}}$ 正比

$$(P,Q,S)_{pu} \text{ on base 2} = (P,Q,S)_{pu} \text{ on base 1} \cdot \frac{S_{base1}}{S_{base2}}$$

$$V_{pu} \text{ on base2} = V_{pu} \text{ on base1} \cdot \frac{V_{base1}}{V_{base2}}$$

$$(R,X,Z)_{pu} \text{ on base2} = (R,X,Z)_{pu} \text{ on base1} \cdot \frac{V_{base1}^2}{V_{base2}^2} \cdot \frac{1}{\frac{S_{base1}}{S_{base2}}}$$

$\frac{V^2}{S}$

例:

$X_{l1}\ (old)=0.02\ (p.u.)$ $X_{l1}\ (new)=?$

$R_1\ (old)=0.05\ (p.u.)$ $R_1\ (new)=?$

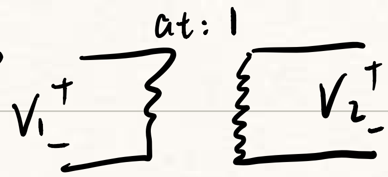
$V_{1base}=500\ V$ $S_{1base}=100mVA$

$V_{2base}=400\ V$ $S_{2base}=500mVA$

$$X_{l1}{}^{new}_{(p.u.)} = 0.02 \times \frac{V_{1base}^2}{V_{2base}^2} \times \frac{S_{2base}}{S_{1base}} = 0.02 \times \frac{500^2}{400^2} \times \frac{500}{100} = 0.156\ (p.u.)$$

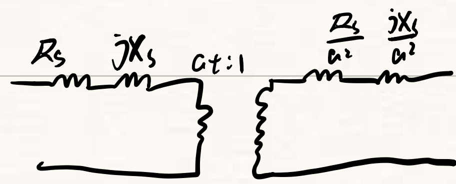
$$R_1{}^{new}_{(p.u.)} = 0.05 \times \left(\frac{500}{400}\right)^2 \times \frac{500}{100} = 0.391\ (p.u.)$$

对于变压器



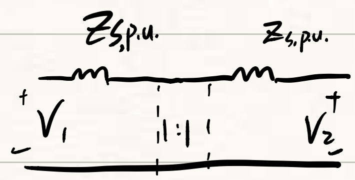
实际 $\frac{V_1}{V_2} = \frac{V_1^{p.u.}}{V_2^{p.u.}} = \frac{V_1^{base}}{V_2^{base}} = a_t$

且有任何情况下副线圈 $Z_{p.u.1} = Z_{p.u.2}$



不受leakage和电阻影响

转换成p.u.时值相同连通



base值代表一整片区域的基准值，只会随变压器比例变化

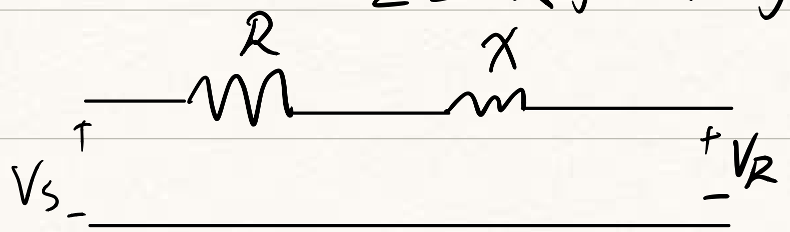
同区域所有元件base值一致，V_{base}^{gen}，V_{base}^{trans}，V_{base}^{dis} 用于所有计算

Transmission Line

Short: 小于80km

单位阻抗

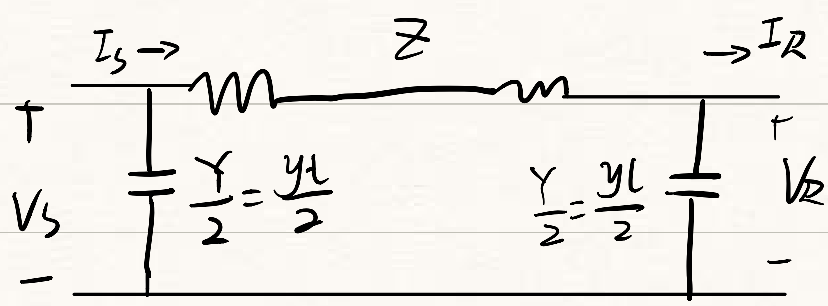
$$Z = (R+j\omega L)\times length$$



$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

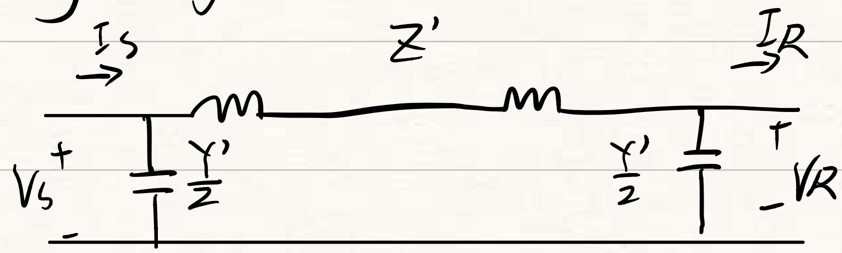
Ybus

Medium Length: 大于80km 小于250km



$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} (1+\frac{YZ}{2}) & Z \\ Y(1+\frac{YZ}{4}) & (1+\frac{YZ}{2}) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Long Length: 大于 250km



$$\frac{Y'}{2} = \frac{Y}{2}\left[\frac{\tanh\frac{\gamma l}{2}}{\frac{\gamma l}{2}}\right] S$$

$$Z' = Z\left[\frac{\sinh(\gamma l)}{\gamma l}\right] \Omega$$

$\gamma = \sqrt{zy}$ Propagation Constant

$Z_c = \sqrt{\frac{z}{y}}$ Impedance